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# Star Formation History of the LMC Bar: "Group 4"

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**Abstract.** I present a few remarks on the measurement of star formation histories and results for the LMC bar.

## 1. Introduction

The "Coimbra experiment" was intended to compare the results of different star formation history codes, as applied to the same data set – something that has not previously been done despite the large number of people working in this field. This contribution describes my technique for measuring star formation histories (especially in ways it differs from other routines), and the results I obtained for the LMC bar.

## 2. Algorithms

My star formation history solution is essentially a three-step process. The first step is the generation of a synthetic CMD for any star formation history, distance, extinction, binary fraction, or IMF. After this is the determination of which synthetic CMD most resembles the observed data and how close it matches. Finally is the determination of an acceptable range of solutions surrounding the best solution, which produces error bars. Each of these steps will be addressed in the sections below. A full description of the technique can be found in Dolphin (2002).

### 2.1. Synthetic CMD Generation

The theory behind synthetic CMD generation is straightforward: the observed CMD is a random drawing from an underlying "model CMD," which itself is a sum of the model CMDs of each of its constituent components (stellar populations of various ages and metallicities, false detections, and foreground stars), which I refer to as "partial CMDs". It is important to note that a model CMD can be generated for any arbitrary star formation history simply by scaling the partial CMDs differently; thus by obtaining a complete set of partial CMDs one can generate any model CMD instantly. In order to use a non-infinite number of partial CMDs, I divide time and metallicity into a finite number of spacings. I have found that $\Delta \log t = 0.1 - 0.2$ and $\Delta[\mathrm{M/H}] = 0.1 - 0.3$ provides enough detail so that most CMDs can be well-fit without having too many free parameters.

Because I am interested in finding the model CMD behind the observed distribution of points, it is critical that the model CMDs not be generated using the common "random population" routine in which a simulated CMD is generated by randomly generating star masses, ages, and metallicities, and placing them on the CMD according to a randomly-chosen artificial star. The error in using such a routine is that it *does not produce a model CMD*; rather it produces a random realization of that model CMD comparable to the data. Because of this, the statistical test becomes one of determining how likely two data sets are to have been drawn from the same unknown underlying model, instead of how likely the observed data is to have been drawn from the model being tested. Not only is this data-data comparison less-constrained than a data-model comparison (thus producing weaker constraints), but *most investigators who work in this field use data-model comparisons with randomly-populated synthetic CMDs.* The $W$ statistic (Saha 1998) is a data-data comparison statistic; a $\chi^2$ statistic or Bayesian inference scheme is not. Because of this mathematics error, most results in this field cannot be trusted.

The correct way to generate a model CMD, unfortunately, is quite time consuming, as one must account for every possible combination of mass, age, metallicity, binarity, and observational effects. While one cannot calculate an infinite number of isochrones, it is critical that the maximum spacing between adjacent isochrones be much smaller than the CMD binning size. This requires very fine interpolation of the isochrones; $\Delta \log t = 0.005$ and $\Delta[\mathrm{M/H}] = 0.02$ will produce isochrones separated by $\sim 0.1$ magnitudes in $V$ and $\sim 0.05$ magnitudes in $(V - I)$. Failing to interpolate will cause severe problems in the solution, as what is being treated as a model CMD does not actually include all possible component populations.

Thus, for each partial CMD, I generate a large number of isochrones to "fill in" the appropriate age and metallicity ranges. Each of these CMDs is then added to the partial CMD, along with binary stars determined according to an assumed binary fraction and mass distribution. Finally, each point in the partial CMD is convolved with the artificial star results, which simulates incompleteness as well as photometric errors. The effective use of artificial stars is another important point. A small number of investigators attempt to model the photometric errors as Gaussian; this is inadequate in terms of accurately modeling observational errors, as incompleteness is a function of the observed magnitude of a star rather than its true magnitude, blending errors depend on the density of stars and the relative distributions within the CMD, and even simple photometric errors are biased and non-Gaussian.

On a side note, I divide the observed CMD into "CMD bins", each of which is rectangular (usually $2 - 3$ times larger in magnitude than in color). Provided this binning is sufficiently fine, there is no loss of accuracy compared with an unbinned solution. However, the unbinned solution schemes I have seen are not capable of accounting for non-Gaussian photometric errors and thus will not adequately model blending and other effects.

I will not go into such detail, but it is also important to have model distributions for bad detections and for foreground stars, which can be added to the other partial CMDs to create the full model CMD. This procedure is clearly superior to the more commonly-used "statistical subtraction", as the subtraction process inevitably leaves residuals (oversubtraction and undersubtraction) and thus a CMD that is *not* representative of the underlying star formation history. My solution of the history of the Sagittarius dSph (Dolphin 2002) illustrates that even severe foreground contamination is no problem if treated properly.

### 2.2. Determination of the Best Fit

With a model CMD in hand, I now attempt to determine how likely the observed data is to have been drawn from the model data. The majority of CMD reconstruction papers have used $\chi^2$ fits to determine the best match. While this ratio of difference to uncertainty has an intuitive appeal, use of $\chi^2$ is completely wrong for CMD modeling.

A $\chi^2$ minimization is a maximum-likelihood calculation for a specific case: Gaussian-distributed data with known uncertainties. The number of stars falling in a CMD bin is not Gaussian-distributed (it is Poisson), nor are the uncertainties known (they go as the square root of the model). In fact, $\chi^2$ *will never correctly determine the most likely model*; it is only a question of how wrong the answer is. Thus one must use a Poisson maximum likelihood calculation, the $\chi^2$ equivalent of which is given by

$$-2 \ln PLR = 2 \sum_i m_i - n_i + n_i \ln \frac{n_i}{m_i}, \tag{1}$$

where $m_i$ is the model CMD in the $ith$ bin and $n_i$ is the observed CMD in that bin. It is worth mentioning that this statistic produces the same star formation history as the Bayesian inference scheme of Tolstoy & Saha (1996); the only difference is that their equation can only be used to determine relative star formation rates at different ages.

A nice feature of using true model CMDs and the Poisson statistics is that local minima do not exist in the solution for the star formation history. The mathematical reason for this is given by Dolphin (2002). The simple answer is that moving from any point towards the "best" point will always improve the fit. This allows for the use of a rapid minimization routine, such as frprmn from *Numerical Recipes* (Press et al. 1992).

A goodness-of-fit can be obtained relatively easily. From the model CMD, one can determine an expectation value for the fit parameter, as well as the variance in values. If one were using $\chi^2$ to fit Gaussian-distributed data, for example, the expectation value would equal the number of degrees of freedom (number of CMD bins minus the number of free parameters) and the variance would equal twice the number of degrees of freedom. It is not as simple for Poisson-distributed data, but the same principle holds. To quantify the fit quality in terms of $\sigma$ away from an ideal fit, I use the following definition:

$$Q = \frac{fit\ parameter - expectation\ value}{\sqrt{variance}}. \tag{2}$$

Since most scientists are more familiar with $\chi^2$ values, I also define an "effective $\chi^2$":

$$\chi^2_{eff} = 1 + Q\sqrt{2/N}, \tag{3}$$

where $N$ equals the number of CMD bins containing either 0.5 or more stars or model points.

### 2.3. Determination of the Uncertainties

As with any scientific measurement, a star formation history has very little value unless error bars are also given. This is not easy, however, as one cannot propagate errors through a minimization routine. Instead, one must determine a range of solutions that produce "acceptable" fits.

Again using the example of Gaussian-distributed data and $\chi^2$, the $1\sigma$ uncertainty space corresponds to all fits within the number of free parameters of the best fit. For example, if a fit used 3 free parameters and was minimized at $\chi^2 = 5.5$, the $1\sigma$ error bars would be set by fits with $\chi^2 = 8.5$. The reason is that the expectation value is 1 for each element in a $\chi^2$ fit, and that each free parameter essentially allows one to perfectly fit one point.

For Poisson-distributed data, the expectation value of the Poisson likelihood ratio varies with the number of model points in each bin. Since the fit is most driven by the bins contributing the most to the variance of the fit parameter, the size of the "acceptable fit" space in an $N$-parameter fit (in terms of the fit parameter) is the sum of the expectation values of the $N$ bins with the largest expected variances. In practice, this sum is slightly more than $N$, as the expectation values slightly exceed 1.0 where the variances are the highest. In this calculation, I set the number of free parameters equal to the number of partial CMDs with nonzero star formation histories, plus two (for the distance and extinction). I do not count all partial CMDs, as some clearly contribute nothing to the model and thus made no effect on the solution.

With the range of acceptable fits defined, one must only move each parameter in the fit until the fit reaches the limit to determine the uncertainty of that parameter. Because adjacent isochrones are nearly-degenerate, it is of course mandatory that the other parameters also be able to vary.

This question of error bars is why I do not use ultra-high resolution in my star formation history solutions. First, the number of free parameters is greatly increased, leading to a larger range of acceptable fits. This factor increases the error bars as the square root of the number of free parameters. Second, each bin contains fewer stars, and therefore can be pushed far from the minimum without much effect on the fit parameter. This, plus the nearly-degenerate nature of adjacent isochrones, increases the error bars proportionally with the number of free parameters. Thus increasing the resolution by 4 in age will increase the error bars by 8.

## 3. Solution of the LMC Bar

Using the principles outlined above and detailed in Dolphin (2002), the solution was run for the LMC bar field. The data used were the HSTphot photometry. The CMD was binned at a fairly high resolution: 0.025 mags $(V-I) \times 0.05$ mags $V$. The analyzed region of the CMD was set by $18.0 \leq V < 24.5$, $16.5 \leq I < 23.5$, and $-0.5 \leq V - I < 2.0$. The upper limits were fixed by incompleteness caused by saturation; the lower limits were set to minimize contamination from poorly-photometered faint stars.

Models were constructed by interpolating Girardi et al. (2000) isochrones, using an assumed IMF slope of $-1.30$, corresponding to a Kroupa (2000) IMF for the star masses in this CMD, and binary fraction of 35% (with a flat secondary mass distribution). The distance and extinction were solved, with a resolution of 0.025 magnitudes each. The star formation history was solved as a two-dimensional function of age and metallicity. Age bins had widths of 0.15 dex; metallicity bins had widths of 0.2 dex over the range $-2.3 \leq$ [M/H] $\leq 0.1$.

The best solution was found using $(m-M)_0 = 18.45$ and $A_V = 0.20$. The fit quality was very good, with $Q = 2.0$ and $\chi^2_{eff} = 1.05$. Fitting the surface x=distance, y=extinction, z=fit quality with a paraboloid, I obtain the following best values and uncertainties from the CMD fit: $(m-M)_0 = 18.44 \pm 0.05$ ($d = 48.8 \pm 1.4$pc) and $A_V = 0.20 \pm 0.06$. Although the star formation rate was determined as a function of both age and metallicity, an easily-tabulated version requires the calculation of the total star formation rate, and mean metallicity measured at each time. These are given in Table 1. A metallicity spread was also measured; it was $\pm 0.1 - 0.3$ dex at ages under 5 Gyr and $\pm 0.4 - 0.5$ dex at older ages.

| Age (Gyr) | SFR[a] | $\langle$[M/H]$\rangle$[b] |
|---|---|---|
| $0.00 - 0.25$ | $1.10^{1.34}_{1.02}$ | $-0.02^{\infty}_{0.18}$ |
| $0.25 - 0.50$ | $2.29^{1.20}_{1.30}$ | $-0.23^{0.11}_{0.10}$ |
| $0.50 - 0.71$ | $1.77^{1.54}_{1.28}$ | $-0.10^{\infty}_{0.14}$ |
| $0.71 - 1.00$ | $1.60^{1.34}_{1.11}$ | $-0.14^{\infty}_{0.17}$ |
| $1.00 - 1.41$ | $1.58^{1.21}_{1.06}$ | $-0.30^{0.13}_{0.16}$ |
| $1.41 - 2.00$ | $2.10^{1.14}_{1.01}$ | $-0.38^{0.14}_{0.12}$ |
| $2.00 - 2.82$ | $2.12^{1.07}_{1.37}$ | $-0.26^{0.16}_{0.13}$ |
| $2.82 - 3.98$ | $0.75^{1.14}_{0.75}$ | $-0.45^{0.23}_{0.32}$ |
| $3.98 - 5.62$ | $1.00^{0.81}_{0.90}$ | $-0.67^{0.16}_{0.20}$ |
| $5.62 - 7.94$ | $0.88^{0.70}_{0.71}$ | $-0.90^{0.32}_{0.31}$ |
| $7.94 - 11.22$ | $0.65^{0.80}_{0.46}$ | $-1.18^{0.41}_{0.43}$ |
| $11.22 - 15.85$ | $0.83^{0.33}_{0.45}$ | $-1.67^{0.20}_{0.30}$ |
| $0.00 - 0.50$ | $1.70^{0.52}_{0.44}$ | $-0.16^{0.09}_{0.09}$ |
| $0.50 - 1.00$ | $1.67^{0.77}_{0.66}$ | $-0.12^{0.11}_{0.11}$ |
| $1.00 - 2.00$ | $1.89^{0.63}_{0.56}$ | $-0.35^{0.11}_{0.11}$ |
| $2.00 - 3.98$ | $1.32^{0.56}_{0.58}$ | $-0.31^{0.09}_{0.17}$ |
| $3.98 - 7.94$ | $0.93^{0.36}_{0.41}$ | $-0.78^{0.20}_{0.27}$ |
| $7.94 - 15.85$ | $0.76^{0.20}_{0.17}$ | $-1.46^{0.17}_{0.22}$ |

[a]Star formation rates are given relative to measured lifetime average rate, $2.71 \pm 0.08 \times 10^{-5} M_{\odot} yr^{-1}$
[b]No upper limit is given for a few metallicity values, because the solution was near the limit of the isochrone set being used.

## 4. Discussion

Some notes regarding the solution technique have been presented. Under the assumption that the observed data are a random realization of a model, much care is taken in producing accurate model CMDs. This involves fine interpolation of isochrones, and the addition of every combination of mass, metallicity, age, binarity, and photometric results; random population of a synthetic CMD is inadequate. The final model CMDs must also contain foreground stars and false detections so that all points can be modeled. The best fit is achieved using Poisson statistics, as $\chi^2$ minimizations are guaranteed to fail when treating Poisson data. The goodness of the fit can be characterized in terms of $\sigma$ away from an ideal solution, or an effective $\chi^2$. Finally, uncertainties are determined by determining that acceptable range of fit parameters.

The results from the LMC bar indicate that the star formation has been continuous from ancient times ($> 11$ Gyr) until the present. The only $> 1\sigma$ feature seen in the star formation rate is that it was definitely below the lifetime average before $\sim 8$ Gyr ago. It appears to have reached a very high star formation rate $\sim 3$ Gyr ago that has been maintained until $\sim 0.25$ Gyr ago, though this cannot be dated accurately due to the error bars. It is possible that there was a lull in star formation from 3 to 4 Gyr ago, though my best solution requires star formation at all ages.

Unlike the rather large error bars for the star formation rates, the metallicity enrichment history is tightly constrained in this solution. The fits require a slightly-higher current metallicity than is generally accepted, although it should be remembered that fits using the Girardi et al. (2000) isochrones generally produce higher metallicities than do spectroscopically-calibrated techniques. That the metallicity enrichment history is so well-constrained is evidence that an "age-metallicity" degeneracy does not exist in CMD synthesis work, at least when enough of the main sequence is present. The recommended enrichment history (from Pagel & Tautvaisiene 1998, Table 1) produced a fit $5\sigma$ worse than the best fit; the problem being that their metallicities are too high for ages of $\sim 5$ Gyr and older and thus placing the RGB too far to the red.

It is encouraging to note that the distance and extinction were both determined very accurately. However, this may make comparison difficult with groups who used the recommended values; both the recommended distance ($(m-M)_0 = 18.50$) and extinction ($A_V = 0.155$) are $1\sigma$ away from the best-fit values. My own attempt to use these values resulted in metallicities $0.05 - 0.1$ dex higher than those in Table 1, as well as a fit $1.5\sigma$ worse than the best fit.

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